COMPUTER SCIENCE DEPARTMENT, UNIVERSITY OF BONN

Knowledge Graph Analysis

Exercise Sheet 4

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1 In Class

1. **Tensors: notation and terminology**
   Identify the tensor entry $\mathcal{T}_{3,5,1}$, the second frontal slice $\mathcal{T}_{:,:,2}$, the firth horizontal slice $\mathcal{T}_{5,:,:}$ and the mode-2 fiber $\mathcal{T}_{4,:,4}$ in the tensor given in figure 1.

2. **Adjacency tensors**
   Draw the knowledge graph described by the adjacency tensor in figure 2

3. **Matrix factorization**
   Given the following matrix,

$$\boldsymbol{X} = \begin{bmatrix} 5 & 3 & 0 & 1 \\ 4 & 0 & 0 & 1 \\ 1 & 1 & 0 & 5 \\ 1 & 0 & 0 & 4 \\ 0 & 1 & 5 & 4 \end{bmatrix} \tag{1.1}$$

   and the following factorization into 2 latent dimensions:

$$\hat{\boldsymbol{X}} = \begin{bmatrix} 2.25 & 0.97 \\ 1.68 & 0.82 \\ 0.26 & 1.97 \\ 0.19 & 1.58 \\ 0.92 & 1.86 \end{bmatrix} \begin{bmatrix} 1.91 & 0.97 & 1.65 & -0.6 \\ 0.82 & 0.42 & 1.78 & 2.44 \end{bmatrix} \tag{1.2}$$

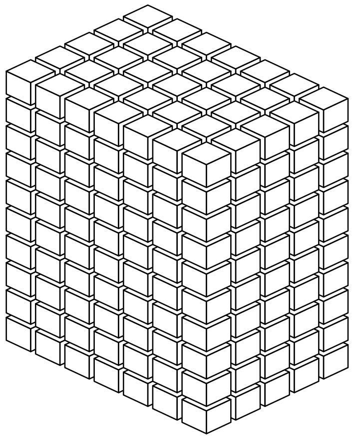

Figure 1.1: Tensor

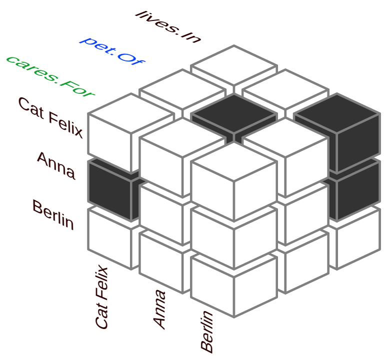

Figure 1.2: Adjacency tensor

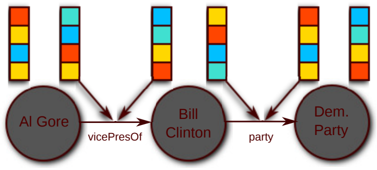

Figure 1.3: Representations learned by a unipartite model

predict the missing (zero) elements of $\boldsymbol{X}$.

4. **Loss function for matrix factorization**
Let

$$\boldsymbol{X} = \begin{bmatrix} 5 & 3 & 0 \\ 4 & 0 & 0 \\ 1 & 1 & 2 \end{bmatrix} \tag{1.3}$$

be a matrix and

$$\boldsymbol{SO}^T = \hat{\boldsymbol{X}} = \begin{bmatrix} 5.1 & 3 & 0.2 \\ 3.9 & 0 & 0.3 \\ 1.1 & 0.9 & 2 \end{bmatrix} \tag{1.4}$$

its approximation gained by matrix factorization. Calculate the Mean Squared Error (MSE) loss

$$||\boldsymbol{X} - \hat{\boldsymbol{X}}||^2 \ . \tag{1.5}$$

5. **Outer product**
Given are the following three vectors

$$\boldsymbol{a} = \begin{bmatrix} 1 & 3 & 2 \end{bmatrix} \tag{1.6}$$

$$\boldsymbol{b} = \begin{bmatrix} 4 & 2 & 1 \end{bmatrix} \tag{1.7}$$

$$\boldsymbol{c} = \begin{bmatrix} 1 & 3 & 4 \end{bmatrix} \tag{1.8}$$

What is the $\boldsymbol{\mathcal{T}}_{:,1,1}$ fiber of the rank-1 tensor $\boldsymbol{\mathcal{T}}$ resulting from the outer product of the vectors

$$\mathcal{T} = \boldsymbol{a} \circ \boldsymbol{b} \circ \boldsymbol{c} \ . \tag{1.9}$$

6. **Collective learning in bipartide model**
Figure 6 shows the latent features/ latent representations learned by an bipartite model (like CP) and Figure 6 shows those learned by a unipartite model (like RESCAL) suitable for collective learning. What is the difference?

7. **RESCAL**

$$\boldsymbol{A} = \begin{bmatrix} 0.1 & 0.8 & 0.1 \\ 0.7 & 0.2 & 0.1 \end{bmatrix} \tag{1.10}$$

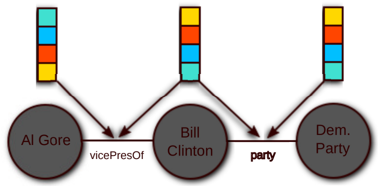

Figure 1.4: Representations learned by a bipartite model

and the k-th frontal slice of the relation tensor

$$\boldsymbol{R}_{:,:,k} = \begin{bmatrix} 1 & 0 & 1.2 \\ 1 & 1.2 & 0 \\ 0.1 & 0 & 1 \end{bmatrix} \tag{1.11}$$

compute the k-th slice of approximated tensor, i.e. $\boldsymbol{A}\boldsymbol{R}_{:,:,k}\boldsymbol{A}^T$. What does the result describe?

8. **Applications of RESCAL**

   a) **Link Prediction**: Let

$$\mathcal{T}_{(1)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{1.12}$$

   be the 1-mode presentation of a tensor and

$$\hat{\mathcal{T}}_{(1)} = \begin{bmatrix} 1.2 & 0.02 & 0.03 & 0.89 & 0.9 & 0.01 & 0.1 & 1.04 & 0.03 \\ 0 & 0.95 & 0.04 & 0.07 & 0.11 & 1.1 & 0.07 & 1.05 & 0.0 \\ 1.1 & 1 & 0.01 & 0.2 & 1.2 & 1 & 0.1 & 0.08 & 0.09 \end{bmatrix} \tag{1.13}$$

   its approximation gained by a tensor decomposition technique. What new links were predicted? What links disappeared?

   b) **Entity Similarity**: Let

$$\boldsymbol{A} = \begin{bmatrix} 0.8 & 0.3 & 1.7 \\ 0.9 & 0.4 & 1.6 \\ 0.2 & 0.2 & 0.3 \\ 1.0 & 0.3 & 1.5 \end{bmatrix} \tag{1.14}$$

   be the entity matrix gained by RESCAL. Which entities are similar?

   c) **Relation Similarity**: Given are the k-th, the l-th, and the m-th frontal slice of the relation tensor of RESCAL

$$\boldsymbol{R}_{:,:,k} = \begin{bmatrix} 1.1 & 0.9 & 0.9 \\ 1.3 & 1.2 & 0.8 \\ 0.9 & 1.1 & 1.1 \end{bmatrix} \tag{1.15}$$

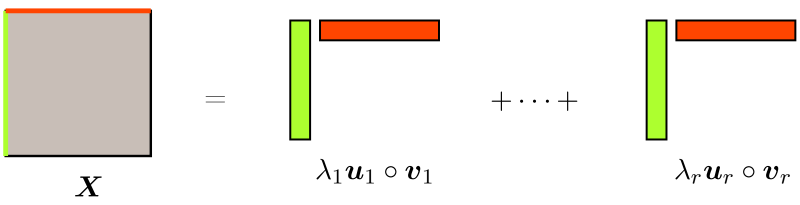

Figure 2.1: SVD

$$\boldsymbol{R}_{:,:,l} = \begin{bmatrix} 2 & 5 & 9 \\ 3 & 0.5 & 1 \\ 0.3 & 4 & 7 \end{bmatrix} \tag{1.16}$$

$$\boldsymbol{R}_{:,:,m} = \begin{bmatrix} 1.2 & 1.1 & 0.8 \\ 1.0 & 0.8 & 0.8 \\ 0.8 & 0.9 & 0.9 \end{bmatrix} \tag{1.17}$$

Which relations are similar?

2 At Home

1. **Singular value decomposition (SVD)**
   For $\boldsymbol{X}$ in (1.1), given are the following set of singular values

$$\lambda_1 = 9.03, \lambda_2 = 6.23, \lambda_3 = 3.77, \lambda_4 = 1.84 \tag{2.1}$$

the left singular vectors

$$\boldsymbol{u}_1 = \begin{bmatrix} -0.44 & -0.3 & -0.52 & -0.4 & -0.54 \end{bmatrix}, \tag{2.2}$$

$$\boldsymbol{u}_2 = \begin{bmatrix} -0.67 & -0.44 & 0.14 & 0.11 & 0.57 \end{bmatrix}, \tag{2.3}$$

$$\boldsymbol{u}_3 = \begin{bmatrix} -0.3 & -0.05 & -0.55 & -0.48 & 0.61 \end{bmatrix}, \tag{2.4}$$

$$\boldsymbol{u}_4 = \begin{bmatrix} -0.49 & 0.8 & -0.29 & 0.21 & 0.08 \end{bmatrix} \tag{2.5}$$

and the right singular vectors:

$$\boldsymbol{v}_1 = \begin{bmatrix} -0.47 & -0.78 & 0.17 & 0.37 \end{bmatrix}, \tag{2.6}$$

$$\boldsymbol{v}_2 = \begin{bmatrix} -0.26 & -0.21 & 0.25 & -0.91 \end{bmatrix}, \tag{2.7}$$

$$\boldsymbol{v}_3 = \begin{bmatrix} -0.3 & 0.46 & 0.81 & 0.21 \end{bmatrix}, \tag{2.8}$$

$$\boldsymbol{v}_4 = \begin{bmatrix} -0.78 & 0.37 & -0.5 & 0 \end{bmatrix} \tag{2.9}$$

compute $\boldsymbol{X}$ as described in figure 1.