

Knowledge Graph Analysis

Exercise Sheet 4

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1 IN CLASS

1. **Tensors: notation and terminology**

Identify the tensor entry $\mathcal{T}_{3,5,1}$, the second frontal slice $\mathcal{T}_{::,2}$, the fifth horizontal slice $\mathcal{T}_{5,::}$ and the mode-2 fiber $\mathcal{T}_{4,::,4}$ in the tensor given in figure 1.1.

2. **Adjacency tensors**

Draw the knowledge graph described by the adjacency tensor in figure 1.2

3. **Converting a Tensor into a Matrix**

Having the tensor shown in 1.3 give the modes

$$X_{(1)}, X_{(2)}, X_{(3)} \tag{1.1}$$

4. **Matrix factorization**

Given the following matrix,

$$\mathbf{X} = \begin{bmatrix} 5 & 3 & 0 & 1 \\ 4 & 0 & 0 & 1 \\ 1 & 1 & 0 & 5 \\ 1 & 0 & 0 & 4 \\ 0 & 1 & 5 & 4 \end{bmatrix} \tag{1.2}$$

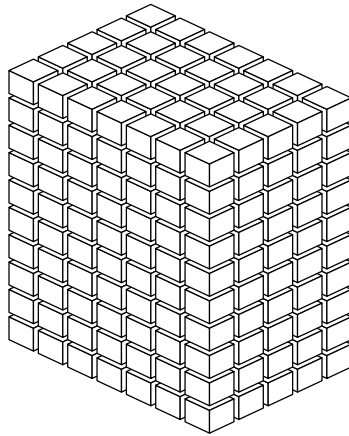


Figure 1.1: Tensor

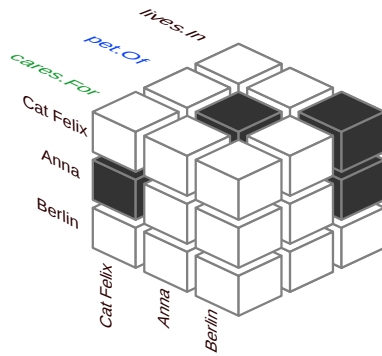


Figure 1.2: Adjacency tensor

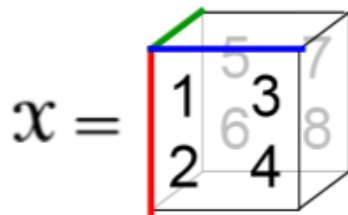


Figure 1.3: Tensor modes

and the following factorization into 2 latent dimensions:

$$\hat{\mathbf{X}} = \begin{bmatrix} 2.25 & 0.97 \\ 1.68 & 0.82 \\ 0.26 & 1.97 \\ 0.19 & 1.58 \\ 0.92 & 1.86 \end{bmatrix} \begin{bmatrix} 1.91 & 0.97 & 1.65 & -0.6 \\ 0.82 & 0.42 & 1.78 & 2.44 \end{bmatrix} \quad (1.3)$$

predict the missing (zero) elements of \mathbf{X} .

5. Loss function for matrix factorization

Let

$$\mathbf{X} = \begin{bmatrix} 5 & 3 & 0 \\ 4 & 0 & 0 \\ 1 & 1 & 2 \end{bmatrix} \quad (1.4)$$

be a matrix and

$$\mathbf{S}\mathbf{O}^T = \hat{\mathbf{X}} = \begin{bmatrix} 5.1 & 3 & 0.2 \\ 3.9 & 0 & 0.3 \\ 1.1 & 0.9 & 2 \end{bmatrix} \quad (1.5)$$

its approximation gained by matrix factorization. Calculate the Mean Squared Error (MSE) loss

$$\|\mathbf{X} - \hat{\mathbf{X}}\|^2 . \quad (1.6)$$

6. Outer product

Given are the following three vectors

$$\mathbf{a} = [1 \quad 3 \quad 2] \quad (1.7)$$

$$\mathbf{b} = [4 \quad 2 \quad 1] \quad (1.8)$$

$$\mathbf{c} = [1 \quad 3 \quad 4] \quad (1.9)$$

What is the $\mathcal{T}_{:,1,1}$ fiber of the rank-1 tensor \mathcal{T} resulting from the outer product of the vectors

$$\mathcal{T} = \mathbf{a} \circ \mathbf{b} \circ \mathbf{c} . \quad (1.10)$$

7. Collective learning in bipartite model

Figure 1.5 shows the latent features/ latent representations learned by an bipartite model (like CP) and Figure 1.4 shows those learned by a unipartite model (like RESCAL) suitable for collective learning. What is the difference?

8. RESCAL

A)

$$\mathbf{A} = \begin{bmatrix} 0.1 & 0.8 & 0.1 \\ 0.7 & 0.2 & 0.1 \end{bmatrix} \quad (1.11)$$

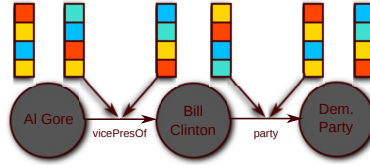


Figure 1.4: Representations learned by a unipartite model

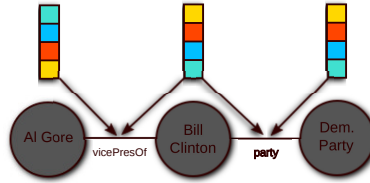


Figure 1.5: Representations learned by a bipartite model

and the k -th frontal slice of the relation tensor

$$\mathbf{R}_{:, :, k} = \begin{bmatrix} 1 & 0 & 1.2 \\ 1 & 1.2 & 0 \\ 0.1 & 0 & 1 \end{bmatrix} \quad (1.12)$$

compute the k -th slice of approximated tensor, i.e. $\mathbf{A}\mathbf{R}_{:, :, k}\mathbf{A}^T$. What does the result describe?

B) Factorization of an adjacency tensor \mathbf{X} using the Rescal model.

Given the adjacency Tensor shown in 1.2 give the components of $\mathbf{X} = \mathbf{A}\mathbf{R}_{i, j, k}\mathbf{A}^T$ What would be the interpretation of estimating the joint distribution over all possible relationships ?

9. Applications of RESCAL

a) **Link Prediction:** Let

$$\mathcal{T}_{(1)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (1.13)$$

be the 1-mode presentation of a tensor and

$$\hat{\mathcal{T}}_{(1)} = \begin{bmatrix} 1.2 & 0.02 & 0.03 & 0.89 & 0.9 & 0.01 & 0.1 & 1.04 & 0.03 \\ 0 & 0.95 & 0.04 & 0.07 & 0.11 & 1.1 & 0.07 & 1.05 & 0.0 \\ 1.1 & 1 & 0.01 & 0.2 & 1.2 & 1 & 0.1 & 0.08 & 0.09 \end{bmatrix} \quad (1.14)$$

its approximation gained by a tensor decomposition technique. What new links were predicted? What links disappeared?

b) **Entity Similarity:** Let

$$\mathbf{A} = \begin{bmatrix} 0.8 & 0.3 & 1.7 \\ 0.9 & 0.4 & 1.6 \\ 0.2 & 0.2 & 0.3 \\ 1.0 & 0.3 & 1.5 \end{bmatrix} \quad (1.15)$$

be the entity matrix gained by RESCAL. Which entities are similar?

c) **Relation Similarity:** Given are the k -th, the l -th, and the m -th frontal slice of the relation tensor of RESCAL

$$\mathbf{R}_{::,k} = \begin{bmatrix} 1.1 & 0.9 & 0.9 \\ 1.3 & 1.2 & 0.8 \\ 0.9 & 1.1 & 1.1 \end{bmatrix} \quad (1.16)$$

$$\mathbf{R}_{::,l} = \begin{bmatrix} 2 & 5 & 9 \\ 3 & 0.5 & 1 \\ 0.3 & 4 & 7 \end{bmatrix} \quad (1.17)$$

$$\mathbf{R}_{::,m} = \begin{bmatrix} 1.2 & 1.1 & 0.8 \\ 1.0 & 0.8 & 0.8 \\ 0.8 & 0.9 & 0.9 \end{bmatrix} \quad (1.18)$$

Which relations are similar?

2 AT HOME

1. Singular value decomposition (SVD)

For \mathbf{X} in (1.2), given are the following set of singular values

$$\lambda_1 = 9.03, \lambda_2 = 6.23, \lambda_3 = 3.77, \lambda_4 = 1.84 \quad (2.1)$$

the left singular vectors

$$\mathbf{u}_1 = [-0.44 \quad -0.3 \quad -0.52 \quad -0.4 \quad -0.54], \quad (2.2)$$

$$\mathbf{u}_2 = [-0.67 \quad -0.44 \quad 0.14 \quad 0.11 \quad 0.57], \quad (2.3)$$

$$\mathbf{u}_3 = [-0.3 \quad -0.05 \quad -0.55 \quad -0.48 \quad 0.61], \quad (2.4)$$

$$\mathbf{u}_4 = [-0.49 \quad 0.8 \quad -0.29 \quad 0.21 \quad 0.08] \quad (2.5)$$

and the right singular vectors:

$$\mathbf{v}_1 = [-0.47 \quad -0.78 \quad 0.17 \quad 0.37], \quad (2.6)$$

$$\mathbf{v}_2 = [-0.26 \quad -0.21 \quad 0.25 \quad -0.91], \quad (2.7)$$

$$\mathbf{v}_3 = [-0.3 \quad 0.46 \quad 0.81 \quad 0.21], \quad (2.8)$$

$$\mathbf{v}_4 = [-0.78 \quad 0.37 \quad -0.5 \quad 0] \quad (2.9)$$

compute \mathbf{X} as described in figure 2.1.

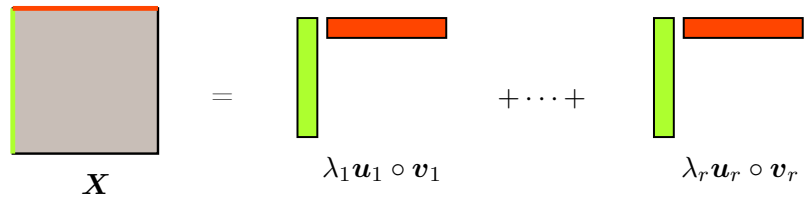


Figure 2.1: SVD

2. Programming Exercise

Check the Exercises in https://github.com/SmartDataAnalytics/Knowledge-Graph-Analysis-Project/tree/master/Exercise_04